

Spin determination of single-produced resonances at hadron colliders

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Credits

- Disclaimer: analysis developed by authors publicly **within CMS** but does **not represent official statement of CMS** about its reach

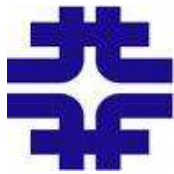
”Spin determination of single-produced resonances at hadron colliders”

arXiv:1001.3396 [hep-ph] (Jan. 19, 2010) \Rightarrow PRD81,075022(2010)

Y.Gao^{1,2,3,4}, A.G.^{1,3,4}, Z.Guo^{1,3,4}, K.Melnikov¹, M.Schulze¹, N.Tran^{1,3}



¹ JHU



² now at FNAL



³ CMS



⁴ BABAR

- Another paper later (some of our CMS colleagues):

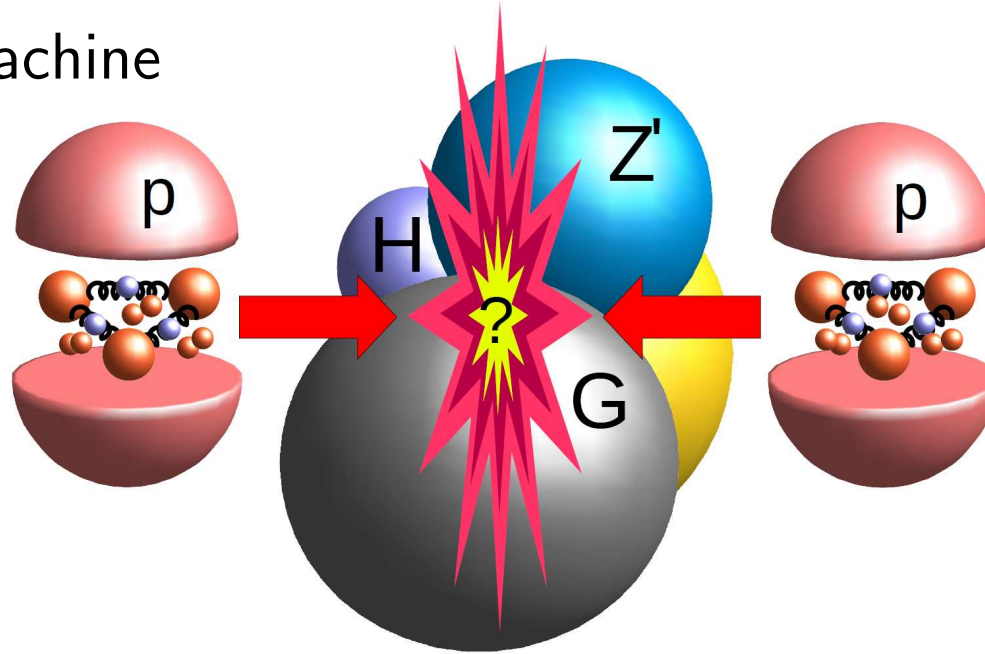
”Higgs look-alikes at the LHC” arXiv:1001.5300 [hep-ph] (Jan. 29, 2010)

A. De Rujula, J. Lykken, M. Pierini, C. Rogan, M. Spiropulu

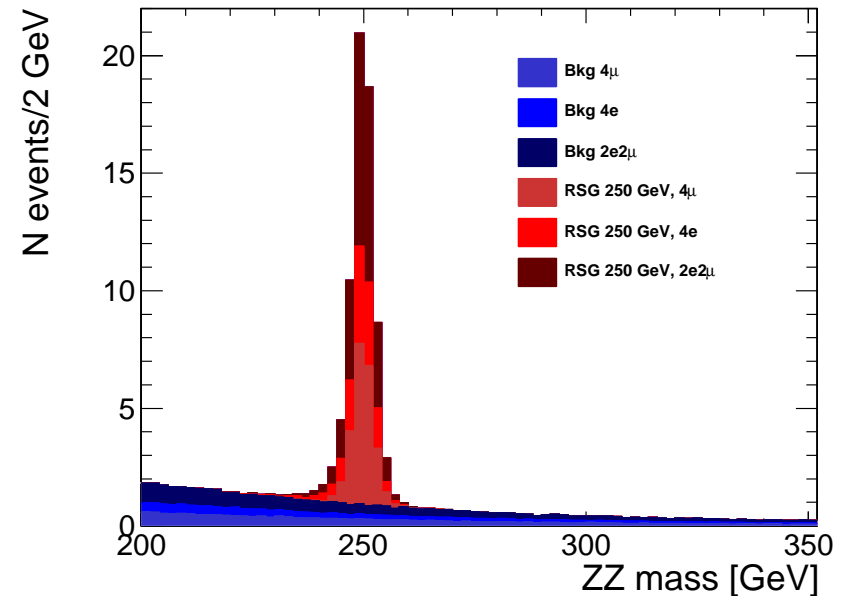
(see next talk)

Questions

- LHC is a discovery machine

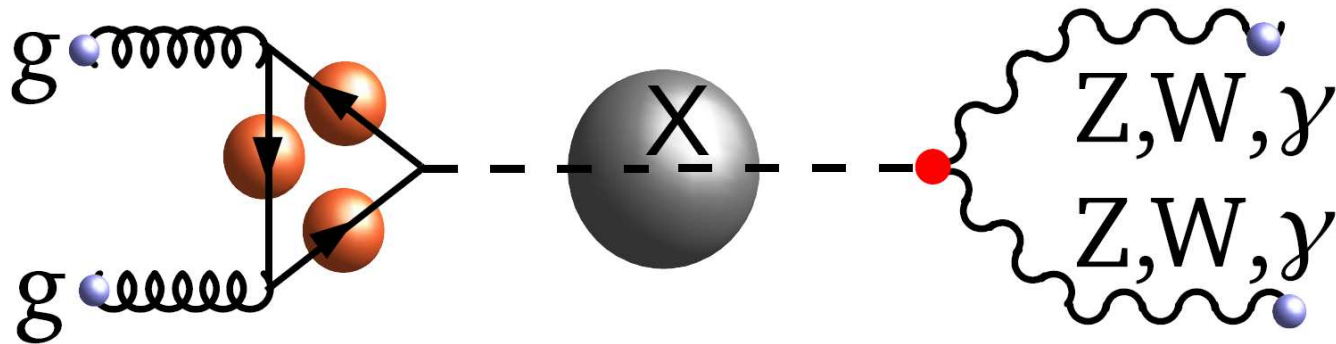


- If resonance is observed on LHC
 - **significance** over background
 - **mass, width, rate** (σ), **branching** can “see” on the mass plot(s) \rightarrow
 - **quantum numbers** (spin, parity,..)?
 - **couplings** to SM fields?
 - **maximum information**?



Higgs Discovery

- Discovery of SM Higgs ($J^P = 0^+$): $H \rightarrow \gamma\gamma, ZZ^{(*)}, W^+W^-, \dots$



- However, other color-neutral & charge-neutral X resonances possible
 - final states (branchings), rate, width differentiate models
 - polarization of $X \rightarrow P_1 P_2 \Rightarrow$ spin & couplings (model-independent)

$$A(H_{J=0} \rightarrow V_1 V_2) = v^{-1} \epsilon_1^{*\mu} \epsilon_2^{*\nu} \left(a_1 g_{\mu\nu} M_X^2 + a_2 q_\mu q_\nu + a_3 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \right)$$

SM $H \rightarrow ZZ^{(*)}, W^+W^-$ tree-level: $a_1 \neq 0$

SM $H \leftrightarrow \gamma\gamma, gg, (Z\gamma)$ loop-induced: $a_1 = -a_2/2 \neq 0$

Beyond SM: any spin and couplings, e.g. $a_3 \neq 0$ for $J^P = 0^-$

Do we expect such new resonances?

- Spin=0 (Higgs)
 - $J^P = 0^+$ SM $H \rightarrow \gamma\gamma, ZZ^{(*)}, W^+W^-, \dots$
 - $J^P = 0^-$ A multi-Higgs models

- Spin=1 (new gauge boson)
 - KK boson, $Z' \rightarrow l^+l^-, q\bar{q}$ dominant
 - models when ZZ and WW dominate

- Spin ≥ 2 (KK graviton, “hidden glueballs”)

- RS Graviton 2^+ (minimal)

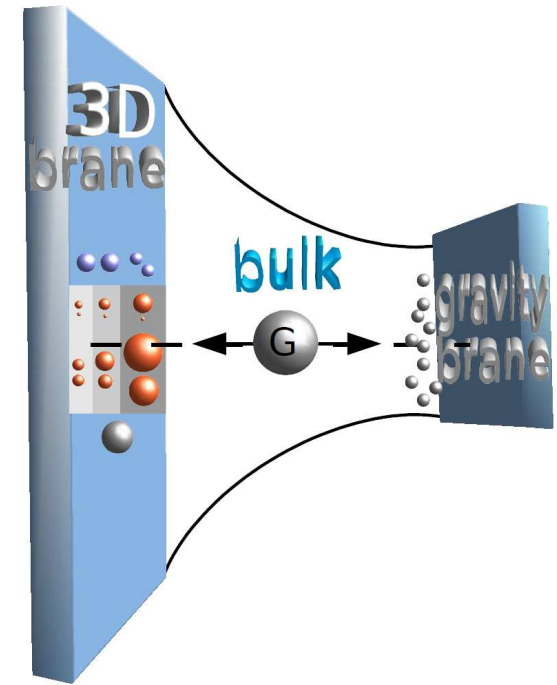
SM on **TeV brane**

$G_{RS} \rightarrow \gamma\gamma$ and l^+l^- discovery

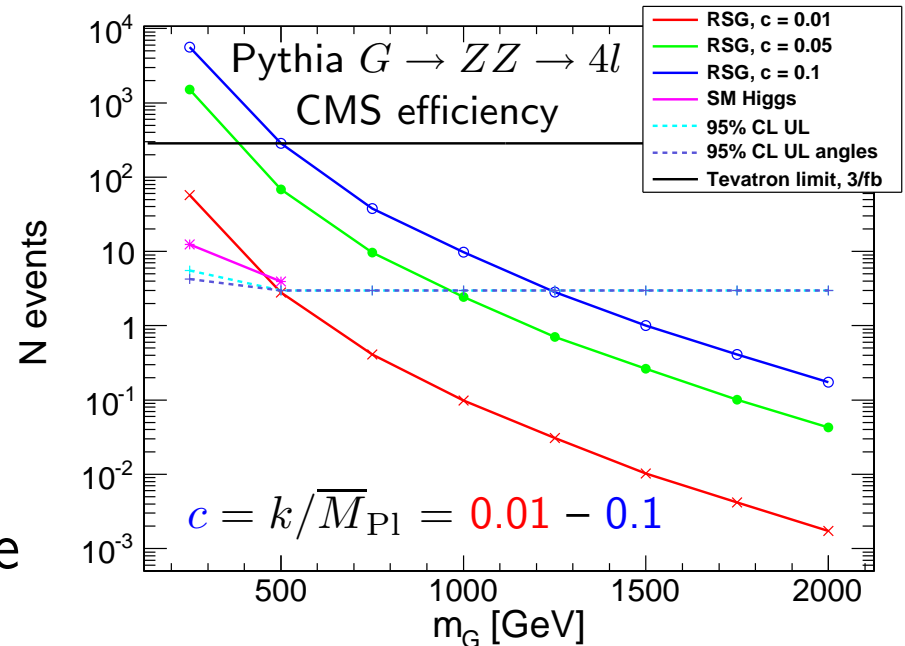
- RS G 2^+ (non-minimal)

light fermions in **bulk**

$G_{RS} \rightarrow W_L^+W_L^-$ and Z_LZ_L dominate

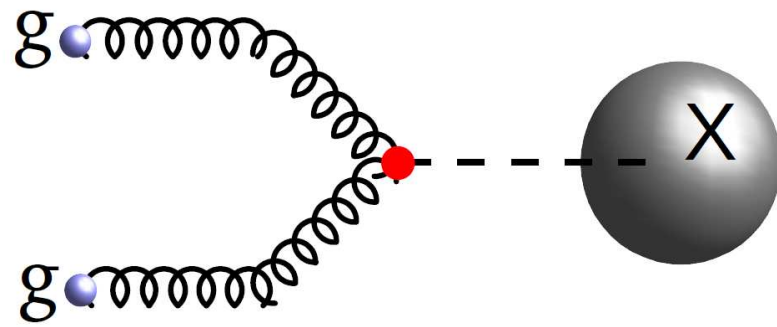


2/fb at $E_{pp} = 14$ TeV



Production of New Resonances

- Consider two dominant production mechanisms



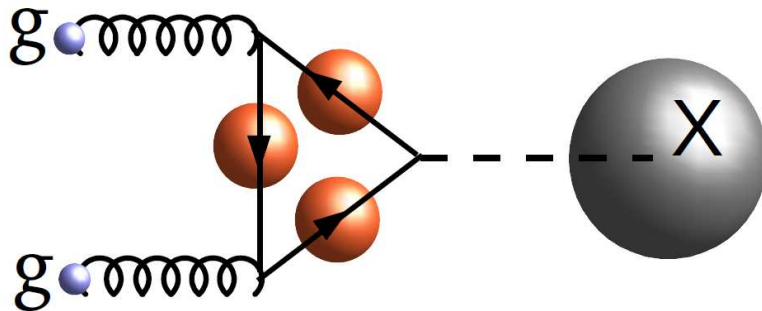
of color-neutral
& charge-neutral X

- Gluon fusion $gg \rightarrow X$

$$J = 0 \text{ or } 2$$

$$J_z = 0 \text{ or } \pm 2$$

expect to dominate at lower mass



- Quark-antiquark $q\bar{q} \rightarrow X$

$$J = 1 \text{ or } 2$$

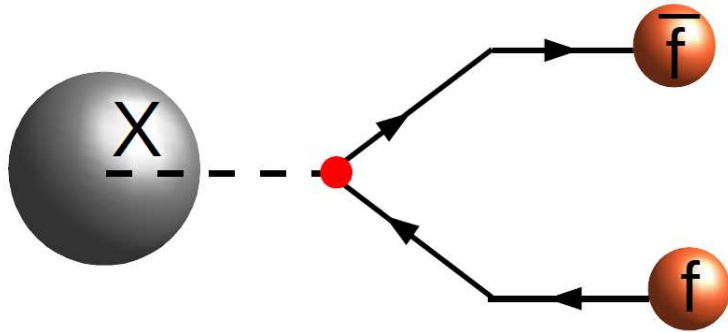
$$J_z = \pm 1 \quad (m_q \rightarrow 0)$$

assume chiral symmetry is exact



Decay of New Resonances

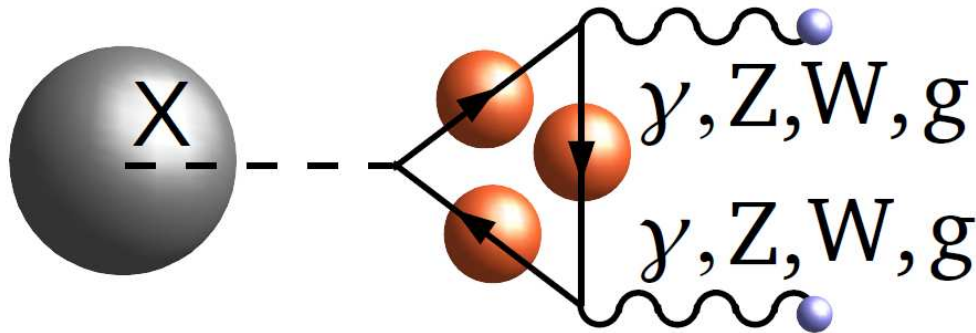
- Consider decay back to Standard Model particles



- Decay to fermions

$$X \rightarrow l^+ l^-, q \bar{q}$$

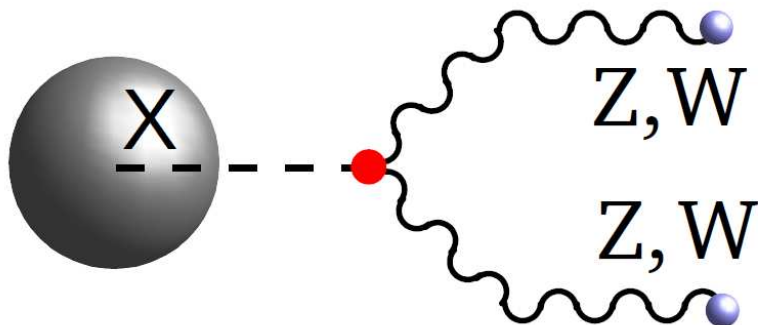
spin-0 excluded $m_f \rightarrow 0$



- Decay to gauge bosons

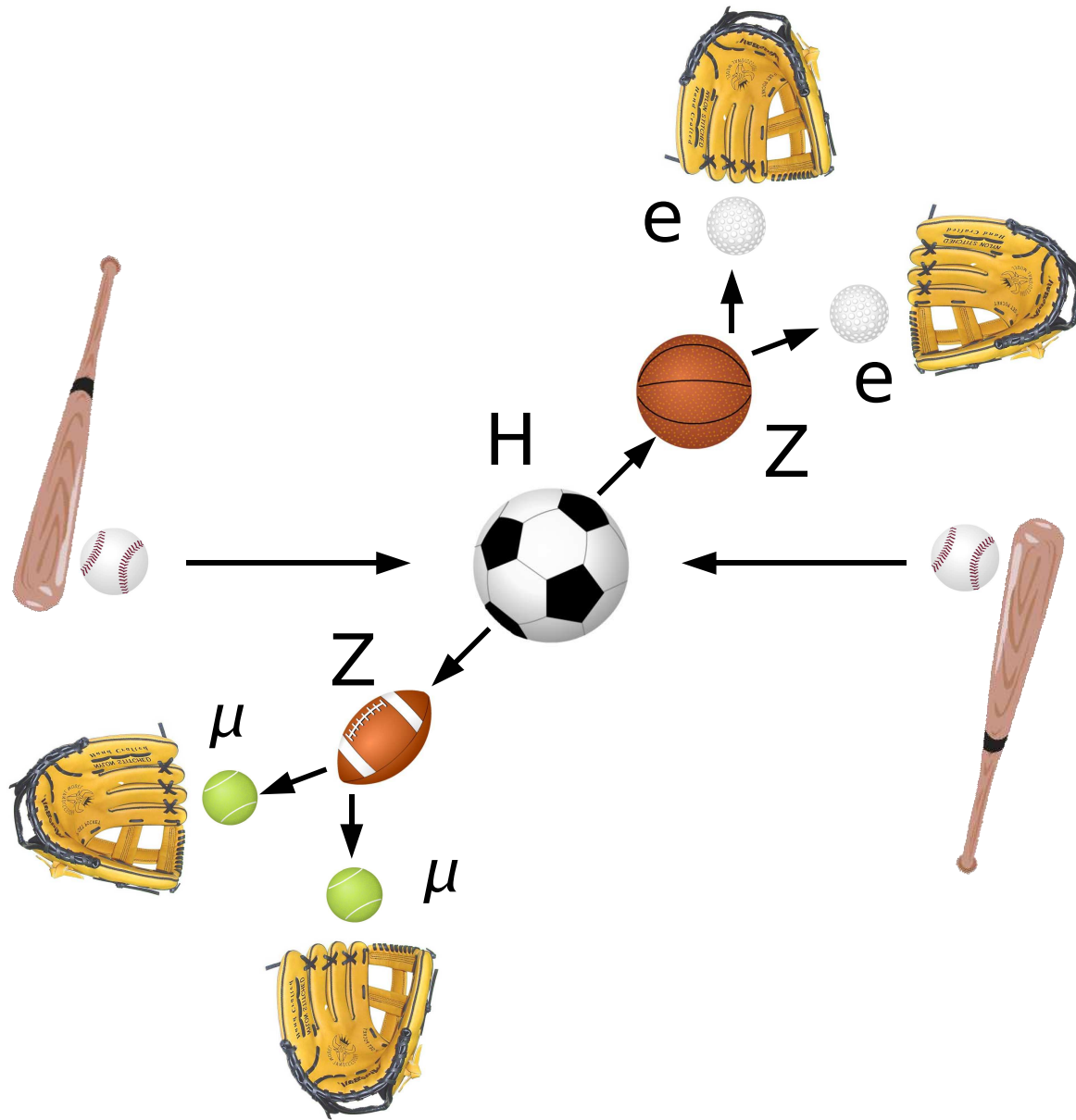
$$X \rightarrow \gamma\gamma, WW, ZZ, Z\gamma, gg$$

spin-1 excluded with $\gamma\gamma, gg$



again X is color-neutral
& charge-neutral

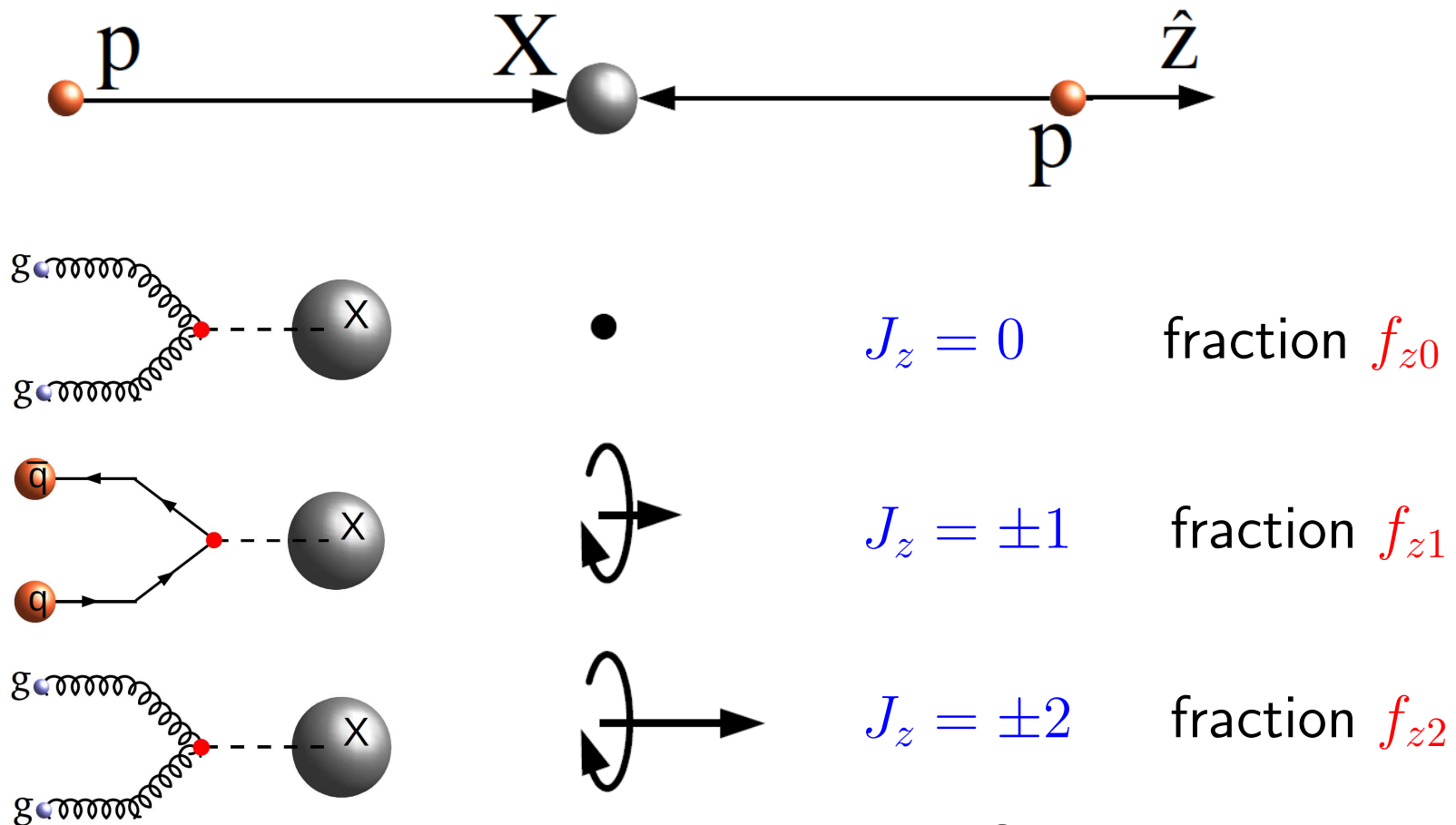
Cartoon of an Experiment



Kinematics in New Resonances Production

- $ab \rightarrow X$ polarization \Leftrightarrow production mechanism and couplings

$$d\sigma_{pp}(\vec{\Omega}) = \sum_{ab} \int dY_X dx_1 dx_2 \tilde{f}_a(x_1) \tilde{f}_b(x_2) \frac{d\sigma_{ab}(x_1 p_1, x_2 p_2, \vec{\Omega})}{dY_X} \Big|_{Y_{ab} = \frac{1}{2} \ln \frac{x_1}{x_2}}$$

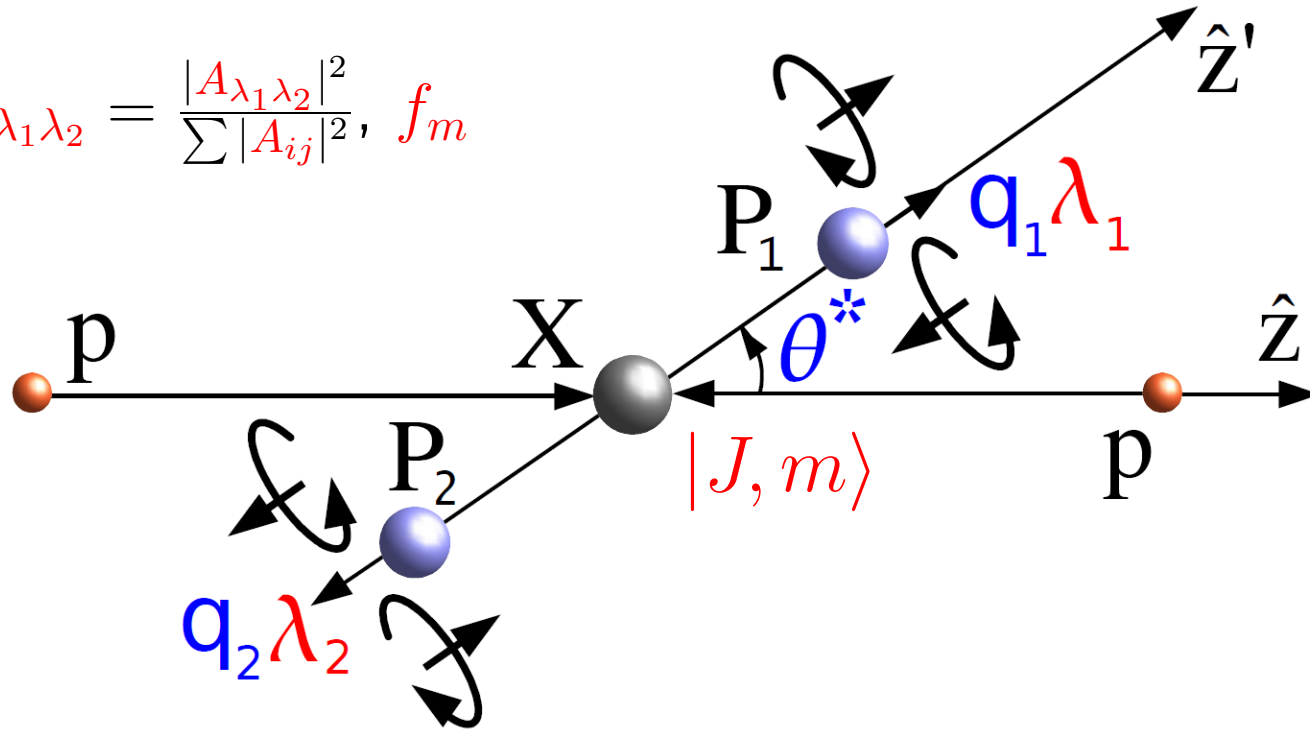


to change f_{z1} : vary LHC energy, sample Y_X

Kinematics in New Resonances Decay

- Only 1 angle θ^* for $X \rightarrow \gamma\gamma, l^+l^-, q\bar{q}, gg$ (but more for ZZ, WW)

fraction $f_{\lambda_1\lambda_2} = \frac{|A_{\lambda_1\lambda_2}|^2}{\sum |A_{ij}|^2}, f_m$



$$\frac{d\Gamma(X_J \rightarrow P_1 P_2)}{\Gamma d \cos \theta^*} = \left(J + \frac{1}{2}\right) \sum_{\lambda_1, \lambda_2} f_{\lambda_1\lambda_2} \sum_m f_m \left(d_{m, \lambda_1 - \lambda_2}^J(\theta^*)\right)^2$$

- Note: if $f_m = \frac{1}{J} \Rightarrow \cos \theta^*$ flat \Rightarrow cannot determine spin
requires f_m fine-tuning (breaks if vary LHC energy, sample Y_X, \dots)

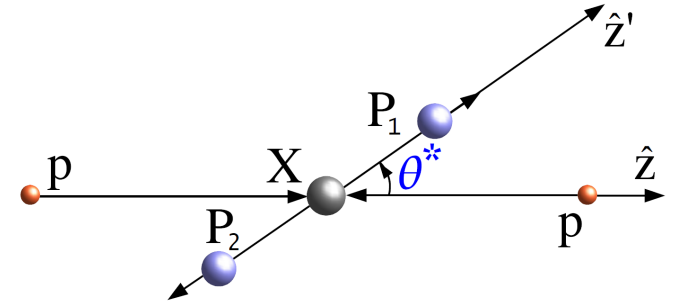
Examples

- if $X \rightarrow \gamma\gamma$ found and $\cos \theta^*$ is flat

(?) spin-0 Higgs \Leftarrow spin-1 excluded

(!) spin-2 not excluded

$\cos \theta^*$ could be flat (but not with min coupling)



$$\frac{16 d\Gamma}{5 \Gamma d \cos \theta^*} = (2 - 2f_{z1} + f_{z2}) - 6(2 - 4f_{z1} - f_{z2}) \cos^2 \theta^* + 3(6 - 10f_{z1} - 5f_{z2}) \cos^4 \theta^* \\ + f_{+-} \left\{ (2 + 2f_{z1} - 7f_{z2}) + 6(2 - 6f_{z1} + f_{z2}) \cos^2 \theta^* - 5(6 - 10f_{z1} - 5f_{z2}) \cos^4 \theta^* \right\}$$

- if $X \rightarrow l^+l^-$ found and $d\Gamma \propto (1 + \cos^2 \theta^*)$

(?) spin-1 Z' \Leftarrow spin-0 excluded

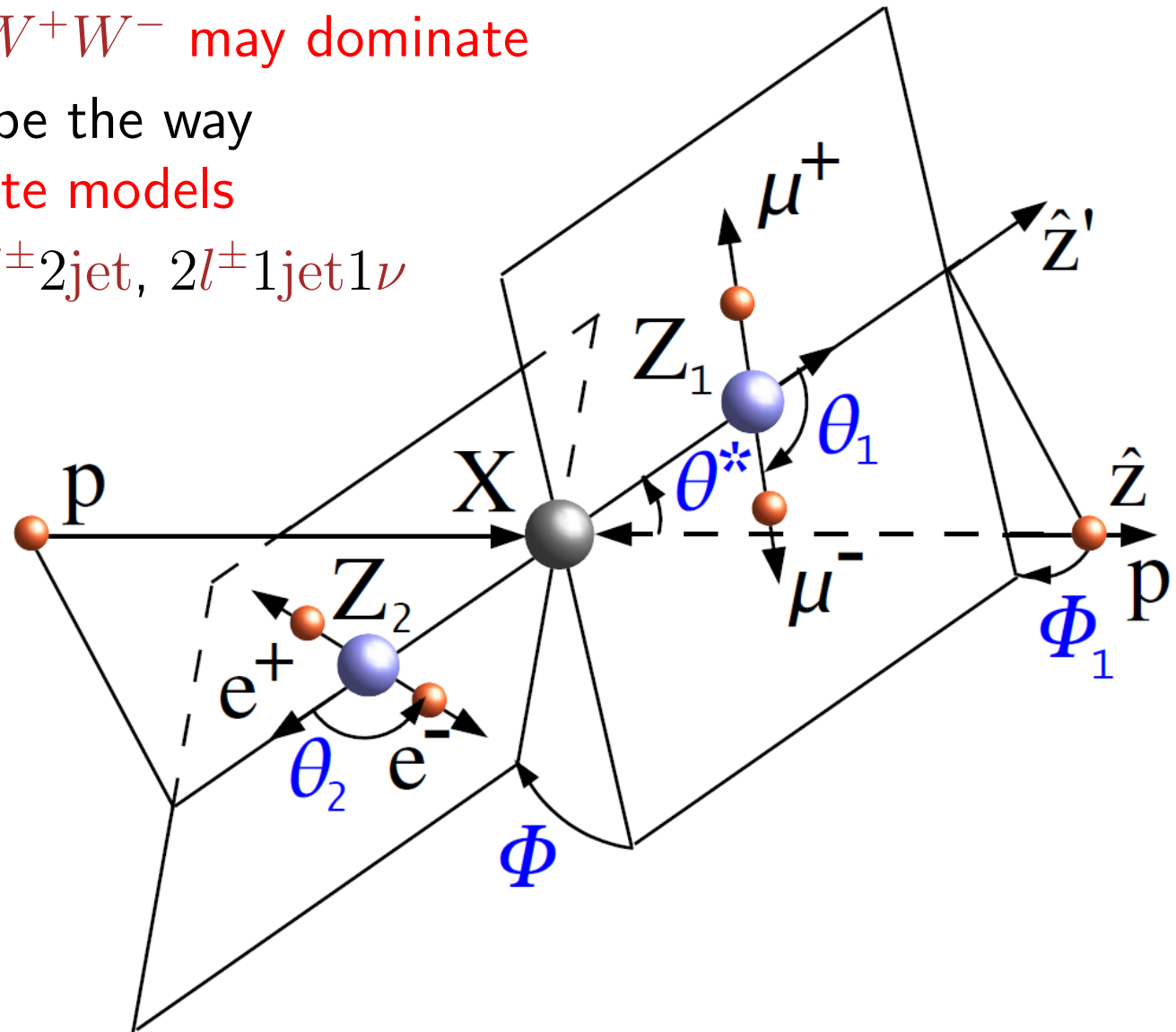
(!) spin-2 not excluded

$\cos \theta^*$ could be the same (but not with min coupling):

$$\frac{16 d\Gamma}{10 \Gamma d \cos \theta^*} = (f_{z1} + f_{z2}) + 3(2 - 3f_{z1} - 2f_{z2}) \cos^2 \theta^* - (6 - 10f_{z1} - 5f_{z2}) \cos^4 \theta^*$$

Kinematics of $X \rightarrow ZZ^{(*)}$ and W^+W^-

- Full information **production** & **decay** angles \Rightarrow multivariate analysis
 - $X \rightarrow ZZ^{(*)}$ & W^+W^- may dominate
 - if not, may still be the way to **differentiate models**
 - full reco: $4l^\pm$, $2l^\pm 2\text{jet}$, $2l^\pm 1\text{jet} 1\nu$

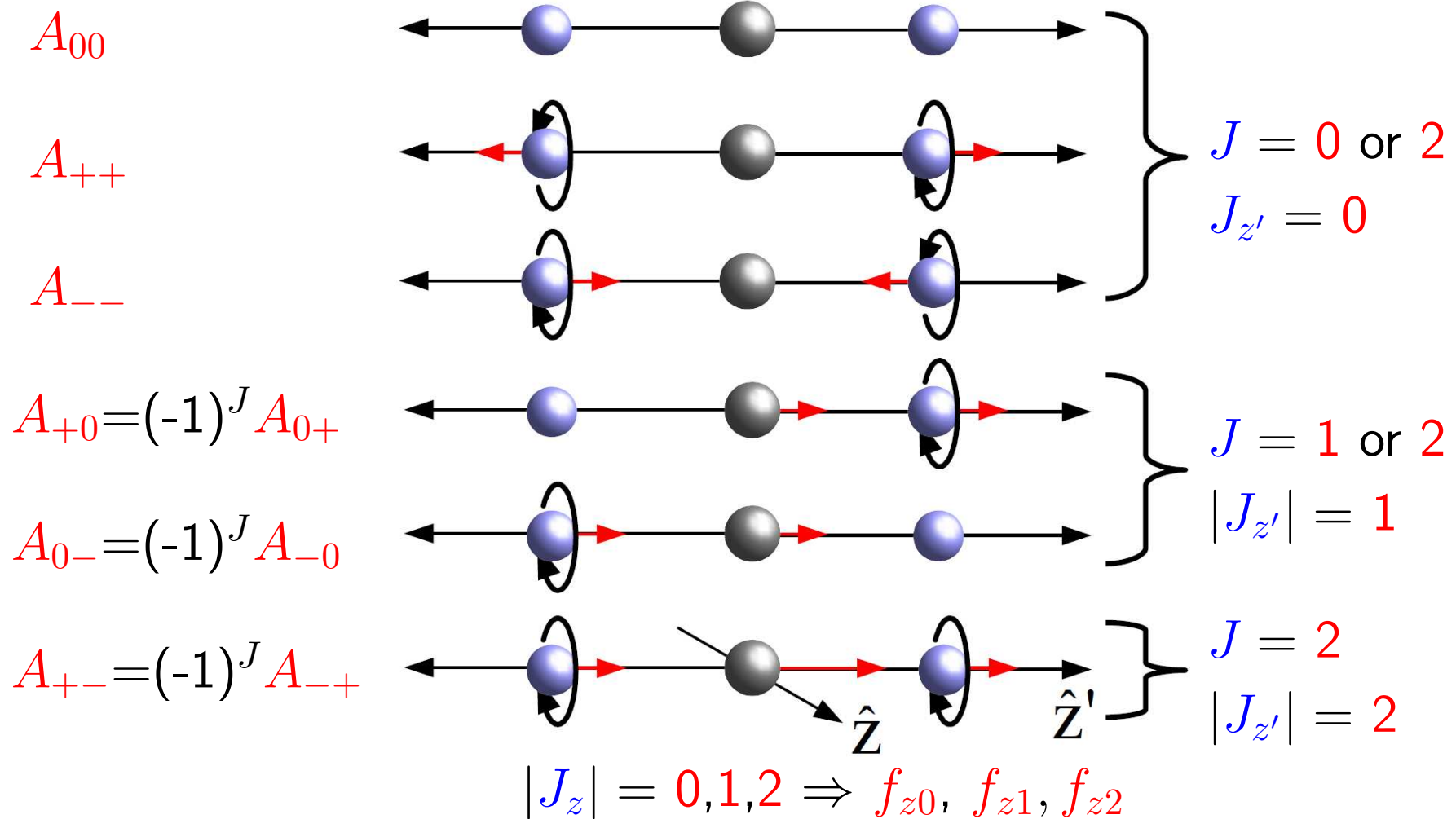


Decay $X \rightarrow ZZ$

- Experimental goal: measure all polarizations (both \hat{z} and \hat{z}')

symmetry in $X \rightarrow ZZ$: $A_{\lambda_1 \lambda_2} = (-1)^J A_{\lambda_2 \lambda_1}$

if parity is a symmetry: $A_{\lambda_1 \lambda_2} = \eta_X (-1)^J A_{-\lambda_1 -\lambda_2}$ (do not use)



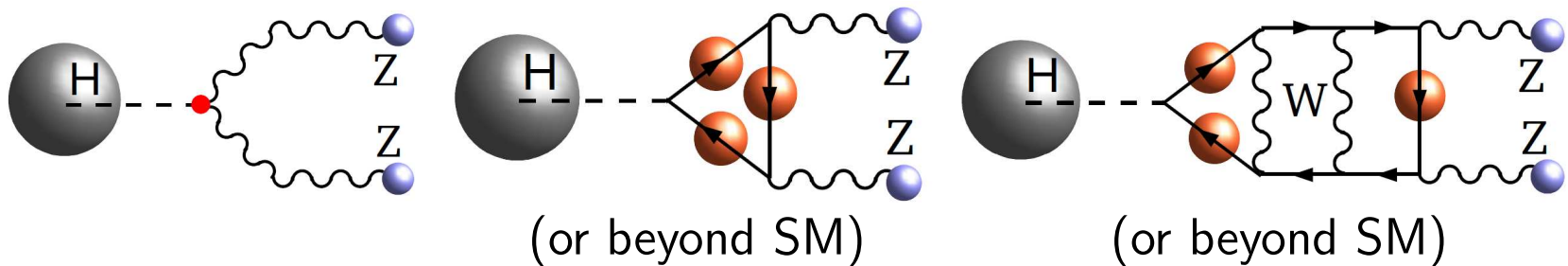
Connect “theory” and “experiment”

Amplitude for Spin-0

- Amplitude for $X_{J=0} \rightarrow V_1 V_2$ (compare $B \rightarrow V_1 V_2$ PRD45,193(1992))

$$A = v^{-1} \epsilon_1^{*\mu} \epsilon_2^{*\nu} \left(a_1 g_{\mu\nu} M_X^2 + a_2 q_\mu q_\nu + a_3 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \right)$$

- SM Higgs 0^+ : (a_1) CP \sim few% (a_2) CP $\sim 10^{-10}$? (a_3) \cancel{CP}



- 3 amplitudes (“experiment”) \Leftrightarrow 3 coupling constants (“theory”)

$$A_{00} = -\frac{M_X^2}{v} \left(a_1 x + a_2 \frac{M_{V_1} M_{V_2}}{M_X^2} (x^2 - 1) \right) \quad \Leftrightarrow \quad x = \frac{M_X^2 - M_{V_1}^2 - M_{V_2}^2}{2M_{V_1} M_{V_2}}$$

$$A_{\pm\pm} = +\frac{M_X^2}{v} \left(a_1 \pm i a_3 \frac{M_{V_1} M_{V_2}}{M_X^2} \sqrt{x^2 - 1} \right)$$

- Below threshold $Z_1 Z_2^*$: $M_{Z_2} < M_{Z_1}$, but generally $a_i(M_{Z_2})$

Higgs Examples

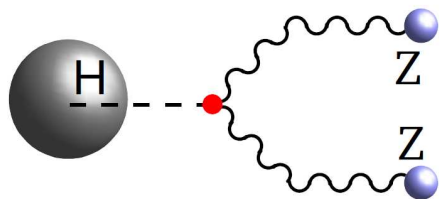
- For a general spin-0 $H \rightarrow V_1 V_2$, define 4 real parameters:

$$f_{\pm\pm} = \frac{|A_{\pm\pm}|^2}{\sum |A_{\lambda_1 \lambda_2}|^2}; \quad \phi_{\pm\pm} = \arg\left(\frac{A_{\pm\pm}}{A_{00}}\right)$$

note: for $H \rightarrow \gamma\gamma, Z\gamma, gg$: $A_{00} \equiv 0 \Rightarrow$ 2 parameters

A_{++} & $A_{--} \Leftrightarrow a_3$ & $a_1 \equiv -a_2/2$, but cannot measure (except $Z\gamma$)

- Assume tree-level SM Higgs decay ($J^P = 0^+$), both ZZ and W^+W^-



$$a_2 = a_3 = 0 \Rightarrow A_{++} = A_{--} = -A_{00} \times \frac{1 - \beta^2}{1 + \beta^2}$$

A_{00} dominates for $M_X \gg M_Z$ (or $\beta \rightarrow 1$)

at $m_H = 250$ GeV: $f_{++} = f_{--} = 0.104$; $\phi_{++} = \phi_{--} = \pi$

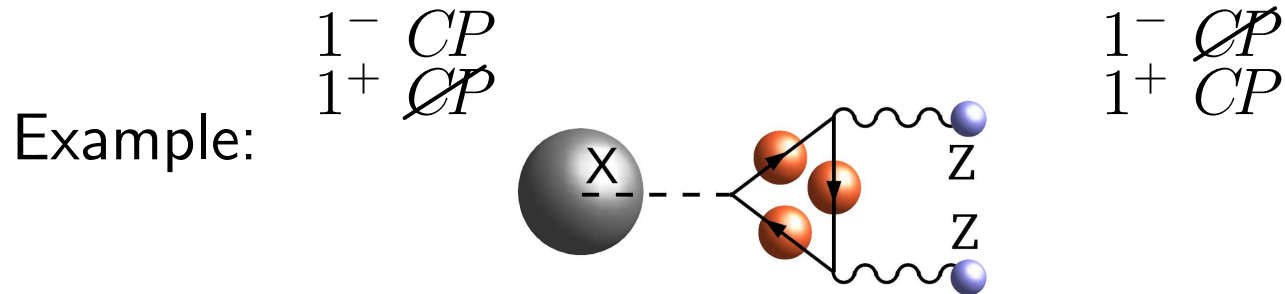
- Pseudoscalar Higgs $A \rightarrow ZZ$ or WW ($J^P = 0^-$): $a_3 \neq 0$

$$f_{++} = f_{--} = 0.500, \quad (\phi_{++} - \phi_{--}) = \pi$$

Amplitude for Spin-1

- Most general amplitude for $X_{J=1} \rightarrow ZZ$

$$A = b_1 [(\epsilon_1^* q_2)(\epsilon_2^* \epsilon_X) + (\epsilon_2^* q_1)(\epsilon_1^* \epsilon_X)] + b_2 \epsilon_{\alpha\mu\nu\beta} \epsilon_X^\alpha \epsilon_1^{*\mu} \epsilon_2^{*\nu} (q_1 - q_2)^\beta$$



- 2 amplitudes (“experiment”) \Leftrightarrow 2 coupling constants (“theory”)

$$A_{+0} \equiv -A_{0+} = \frac{\beta m_X^2}{2m_Z} (b_1 + i\beta b_2)$$

$$A_{-0} \equiv -A_{0-} = \frac{\beta m_X^2}{2m_Z} (b_1 - i\beta b_2)$$

(compare $Z' \rightarrow ZZ$ by C.P.Buszello et al., Eur.Phys.J.C32,209(2004)

W.Y.Keung, I.Low, J.Shu, PRL101,091802(2008)

but consider b_1 and b_2 generally complex and use all angles, see later)

Amplitude for Spin-2

$$\begin{aligned}
 & 2^+ \text{ } CP \\
 & 2^- \text{ } \cancel{CP} \\
 & 2^+ \text{ } \cancel{CP} \\
 & 2^- \text{ } CP \\
 A = & \frac{e_1^{*\mu} e_2^{*\nu}}{\Lambda} \left[c_1 t_{\mu\nu}(q_1 q_2) + c_2 g_{\mu\nu} t_{\alpha\beta} (q_1 - q_2)^\alpha (q_1 - q_2)^\beta \right. \\
 & + \frac{c_3 t_{\alpha\beta}}{M_X^2} q_{2\mu} q_{1\nu} (q_1 - q_2)^\alpha (q_1 - q_2)^\beta + 2c_4 (t_{\mu\alpha} q_{1\nu} q_2^\alpha + t_{\nu\alpha} q_{2\mu} q_1^\alpha) \\
 & + \frac{c_5 t_{\alpha\beta}}{M_X^2} (q_1 - q_2)^\alpha (q_1 - q_2)^\beta \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma + c_6 t^{\alpha\beta} (q_1 - q_2)_\beta \epsilon_{\mu\nu\alpha\rho} q^\rho \\
 & \left. + \frac{c_7 t^{\alpha\beta}}{M_X^2} (q_1 - q_2)_\beta (\epsilon_{\alpha\mu\rho\sigma} q^\rho (q_1 - q_2)^\sigma q_\nu + \epsilon_{\alpha\nu\rho\sigma} q^\rho (q_1 - q_2)^\sigma q_\mu) \right]
 \end{aligned}$$

- 6 amplitudes (“experiment”) \Leftrightarrow 6 combinations of coupl. const.

$$A_{00} = \frac{M_X^4}{M_V^2 \sqrt{6} \Lambda} \left[(1 + \beta^2) \left(\frac{c_1}{8} - \frac{c_2}{2} \beta^2 \right) - \beta^2 \left(\frac{c_3}{2} \beta^2 - c_4 \right) \right]$$

$$A_{\pm\pm} = \frac{M_X^2}{\sqrt{6} \Lambda} \left[\frac{c_1}{4} (1 + \beta^2) + 2c_2 \beta^2 \pm i\beta (c_5 \beta^2 - 2c_6) \right]$$

$$A_{\pm 0} \equiv A_{0\pm} = \frac{M_X^3}{M_V \sqrt{2} \Lambda} \left[\frac{c_1}{8} (1 + \beta^2) + \frac{c_4}{2} \beta^2 \mp i\beta \frac{(c_6 + c_7 \beta^2)}{2} \right]$$

$$A_{+-} \equiv A_{-+} = \frac{M_X^2}{4\Lambda} c_1 (1 + \beta^2)$$

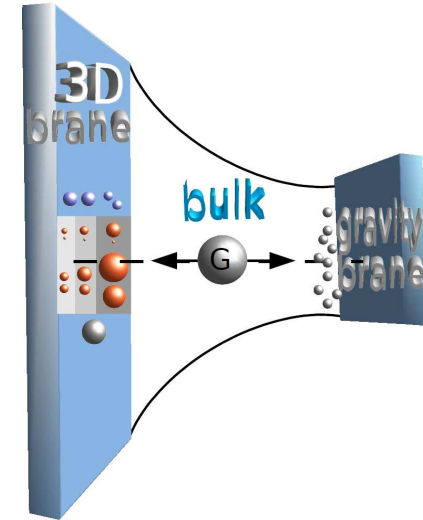
Note about Graviton couplings

- Minimal G_{RS} coupling:

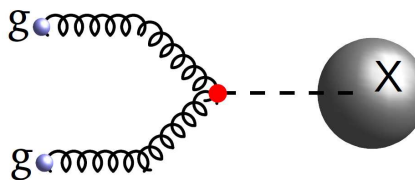
$$A \propto \frac{1}{\Lambda} t_{\mu\nu} \mathcal{T}^{\mu\nu}$$

→ energy-mom tensor → SM field-strength tensor

$$\mathcal{T}_{\mu\nu} = F_{\mu\alpha}^{*(1)} F_{\nu\beta}^{*(2)} g^{\alpha\beta} + m_V^2 \epsilon_1^{*\mu} \epsilon_2^{*\nu} \quad \& \quad F^{(i)\mu\nu} = \epsilon_i^\mu q_i^\nu - \epsilon_i^\nu q_i^\mu$$



- Consequence: $c_2 = \frac{c_4}{2} \simeq -\frac{c_1}{4}$ (as $\beta \rightarrow 1$) $\Rightarrow A_{+-}$ & A_{-+} dominate

 $\Rightarrow gg \rightarrow X$ only $J_z = \pm 2 \Rightarrow f_{z0} = 0 \Rightarrow$ polarized X

$X \rightarrow ZZ$ at $m_G = 250$ GeV: $f_{+-} + f_{-+} = 0.56$, $f_{00} = 0.11$

at $m_G = 1000$ GeV: $f_{+-} + f_{-+} = 0.89$, $f_{00} = 0.11$

- Non-minimal coupling (e.g. SM in the bulk)

generally $A_{00} \propto \frac{M_X^4}{M_V^2 \Lambda}$ dominates, $f_{00} \rightarrow 1.0$ for large M_X

- Notation later: 2_m^+ minimal G ; 2_L^+ longitudinal, like Higgs (!)

Coupling to fermions

- For completeness $X \rightarrow q\bar{q}$, also to describe $q\bar{q} \rightarrow X$:

$$A_{J=0} = \frac{m_q}{v} \bar{u}_{q_1} \left(\rho_3^{(0)} + \rho_4^{(0)} \gamma_5 \right) v_{q_2}$$

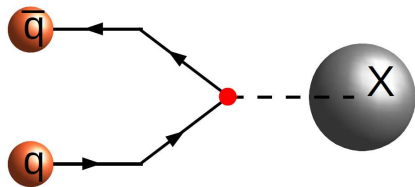
$$A_{J=2} = \frac{1}{\Lambda} t^{\mu\nu} \bar{u}_{q_1} \left(\gamma_\mu \Delta q_\nu (\rho_1 + \rho_2 \gamma_5) + \frac{m_q}{\Lambda^2} \Delta q_\mu \Delta q_\nu (\rho_3 + \rho_4 \gamma_5) \right) v_{q_2}$$

- 4 amplitudes (“experiment”) \Leftrightarrow 4 coupling constants (“theory”)

$$A_{\pm\pm} = \frac{2\sqrt{2} m_q M_X \beta}{\sqrt{3}\Lambda} \left(\pm \rho_1 + \frac{\beta M_X^2}{2\Lambda^2} (\rho_4 \mp \rho_3 \beta) \right)$$

$$A_{\pm\mp} = \frac{M_X^2 \beta}{\Lambda} (\mp \rho_1 - \beta \rho_2)$$

- Consequence of m_q (chiral symmetry)



$$\Rightarrow A_{++} = A_{--} = 0 \text{ at } m_q \rightarrow 0$$

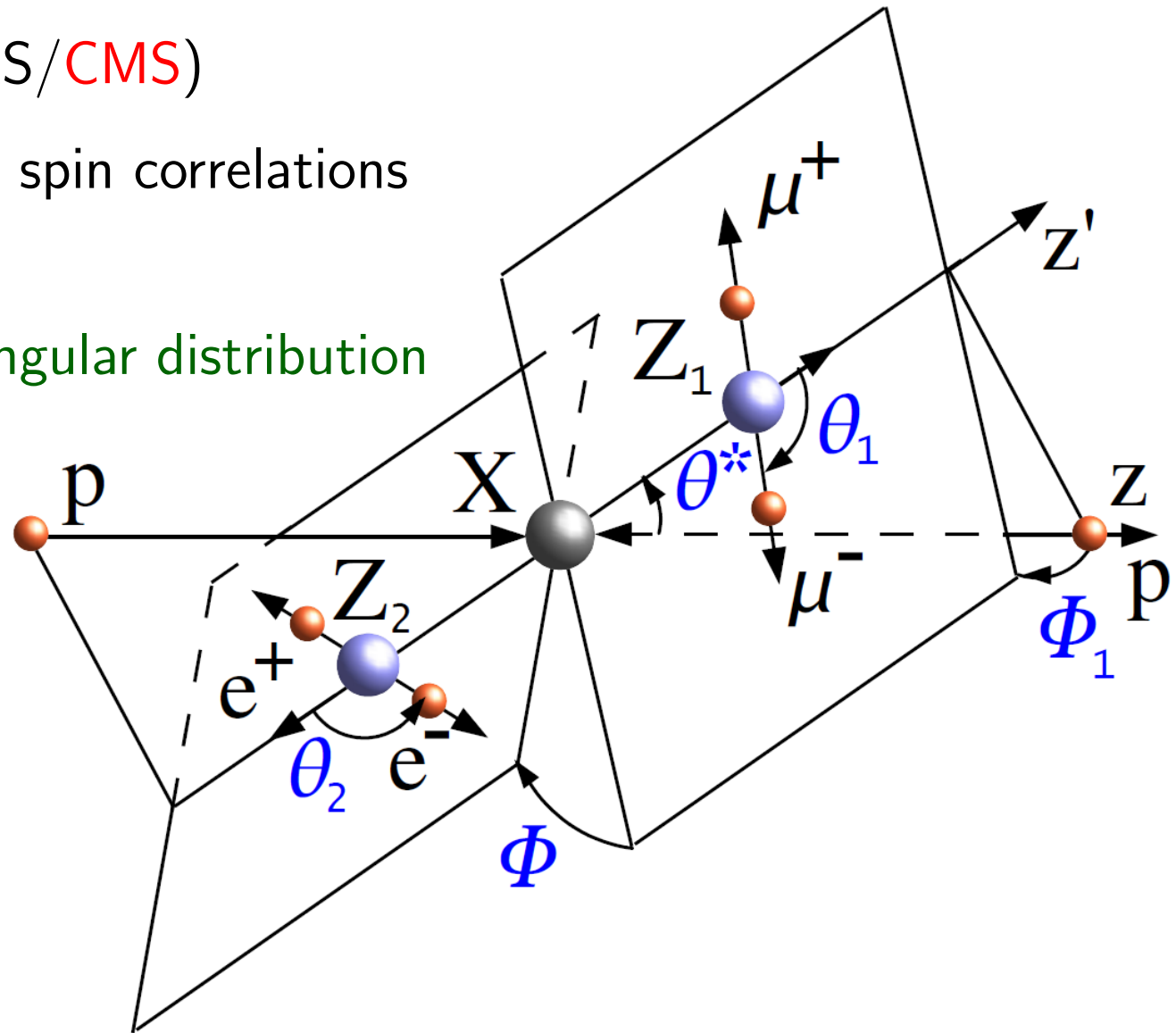
$$\Rightarrow A_{\uparrow\downarrow}, A_{\downarrow\uparrow} \Rightarrow J_z = \pm 1 \text{ in } q\bar{q} \rightarrow X$$

How to measure polarization

How to Measure Polarization

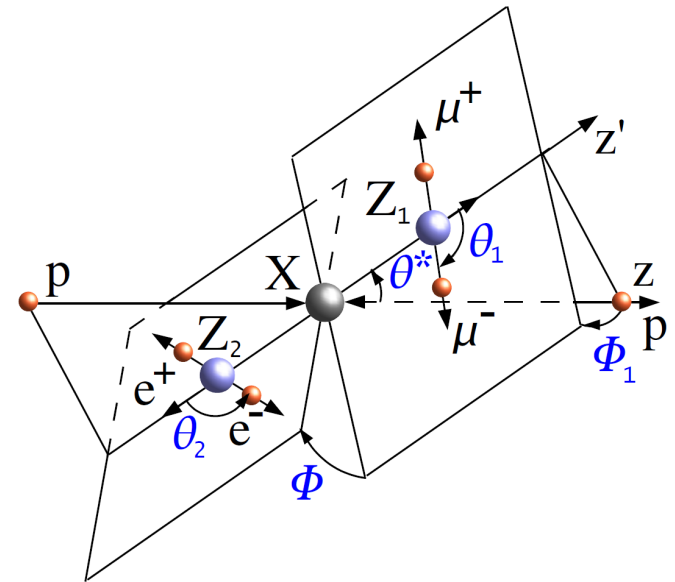
- Deduce all $A_{\lambda_1\lambda_2}$ from angular distributions for **any** J , but need:

- (1) **detector** (ATLAS/CMS)
- (2) full **MC** with all spin correlations
(none before)
- (3) full analytical **angular distribution**
(none before)
- (4) likelihood **fit**
(learn from B 's)



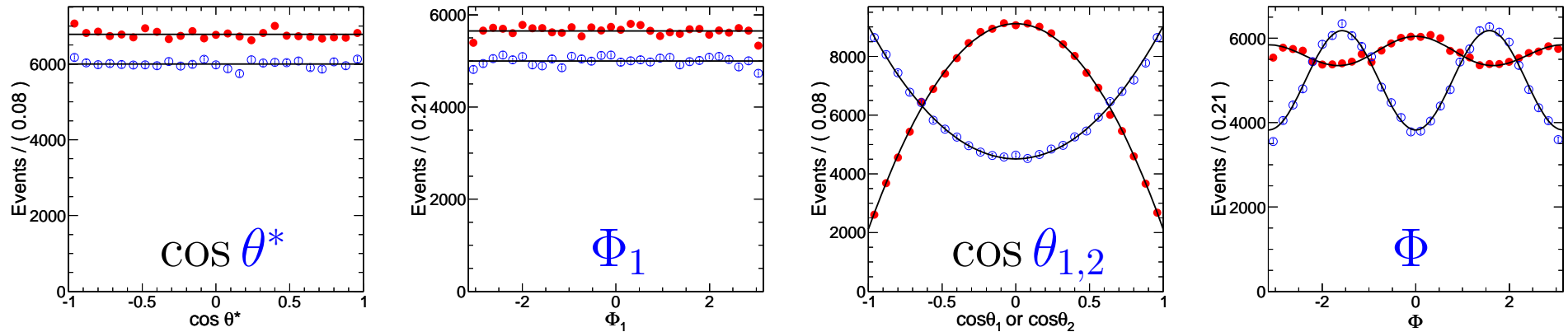
Monte Carlo Simulation

- MC program, open access: <http://www.pha.jhu.edu/spin/>
 - complete kinematic chain (BW) $ab \rightarrow X \rightarrow ZZ \rightarrow (f_1 \bar{f}_1)(f_2 \bar{f}_2)$
 - calculate matrix element $|M|^2$ (narrow-width approximation)
 - weigh or accept/discard events
- Important features:
 - most general couplings for $J = 0, 1, 2$
 - e.g. Higgs radiative corrections
 - e.g. non-minimal G couplings, $Z' \rightarrow ZZ$
 - any angular distribution from QM
 - interface to detector simulation (Pythia)
- Background
 - MadGraph: $q\bar{q} \rightarrow ZZ$ ($gg \rightarrow ZZ \sim 15\%$)
 - others negligible: $Zb\bar{b}$, $t\bar{t}$, $W^+W^-b\bar{b}$, WWZ , $t\bar{t}Z$, $4b$
 l^\pm isolation, $4l$ vertex, $2l$ mass, (no missing energy)...

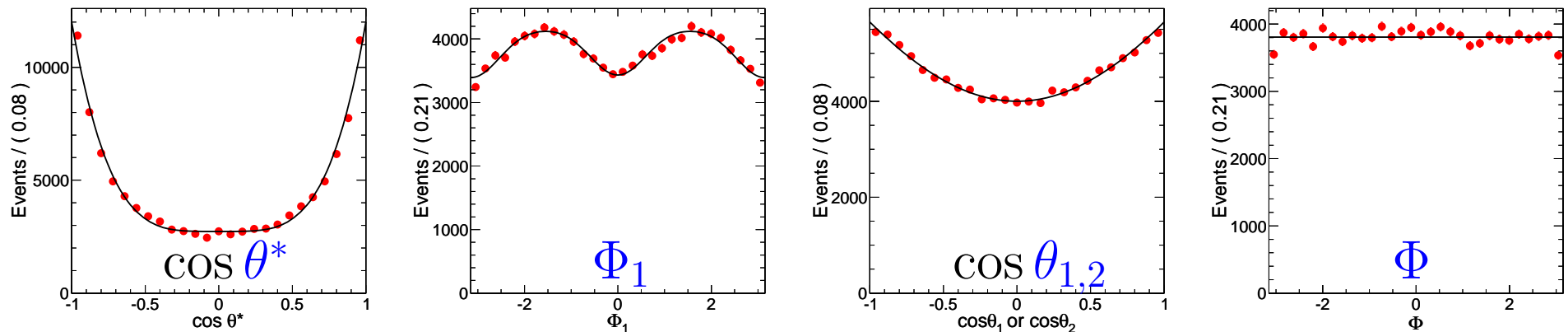


Simulation Examples

- Higgs 0^+ (SM tree-level, a_1) and 0^- (a_3) at $m_H = 250$ GeV
 - lines from derived distributions (independent, next slides)



- Background $q\bar{q} \rightarrow ZZ$
 - lines empirical shape



Angular Distributions

- Connect **amplitudes** and **angular distributions**

for any $J = 0, 1, 2, 3, 4, \dots$ (to cover any “hidden glueball” etc...)

$$A_{ab} \propto D_{\chi_1 - \chi_2, m}^{J*}(\Omega^*) B_{\chi_1 \chi_2} \times D_{m, \lambda_1 - \lambda_2}^{J*}(\Omega) A_{\lambda_1 \lambda_2} \\ \times D_{\lambda_1, \mu_1 - \mu_2}^{S_1*}(\Omega_1) T(\mu_1, \mu_2) \times D_{\lambda_2, \tau_1 - \tau_2}^{S_2*}(\Omega_2) W(\tau_1, \tau_2)$$

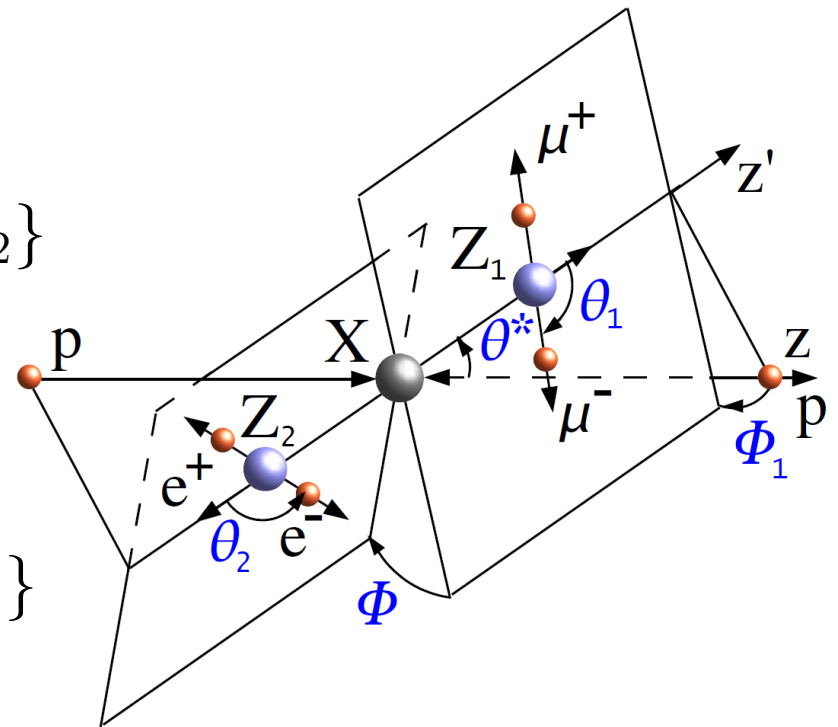
$$d\sigma \propto \sum_{\chi, \mu, \tau} \left| \sum_{\lambda, m} A_{ab}(\{\Omega\}) \right|^2$$

$$ab \rightarrow X, \quad \Omega^* = (\Phi_1, \theta^*, -\Phi_1), \quad \{\chi_1 \chi_2\}$$

$$X \rightarrow Z_1 Z_2^{(*)}, \quad \Omega = (0, 0, 0), \quad \{\lambda_1 \lambda_2\}$$

$$Z_1 \rightarrow f_1 \bar{f}_1, \quad \Omega_1 = (0, \theta_1, 0), \quad \{\mu_1, \mu_2\}$$

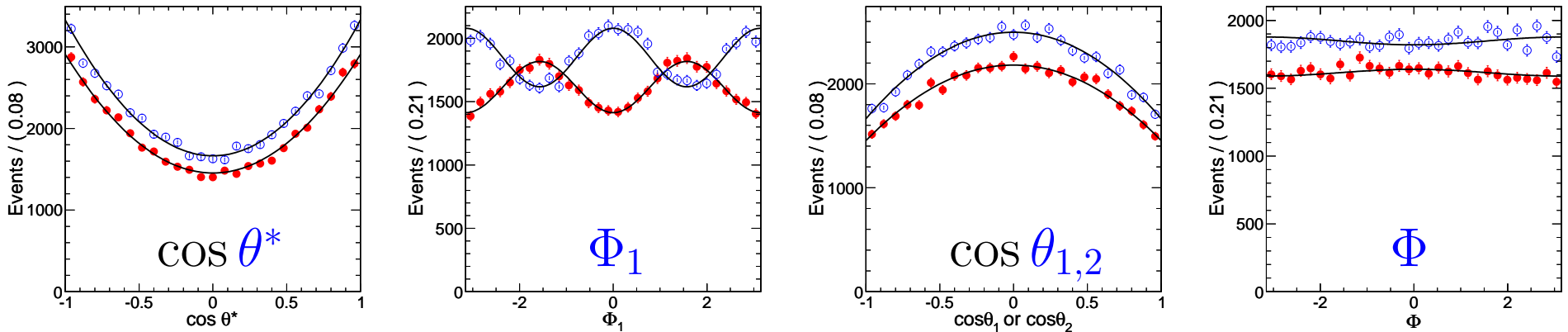
$$Z_2^{(*)} \rightarrow f_2 \bar{f}_2, \quad \Omega_2 = (\Phi, \theta_2, -\Phi), \quad \{\tau_1, \tau_2\}$$



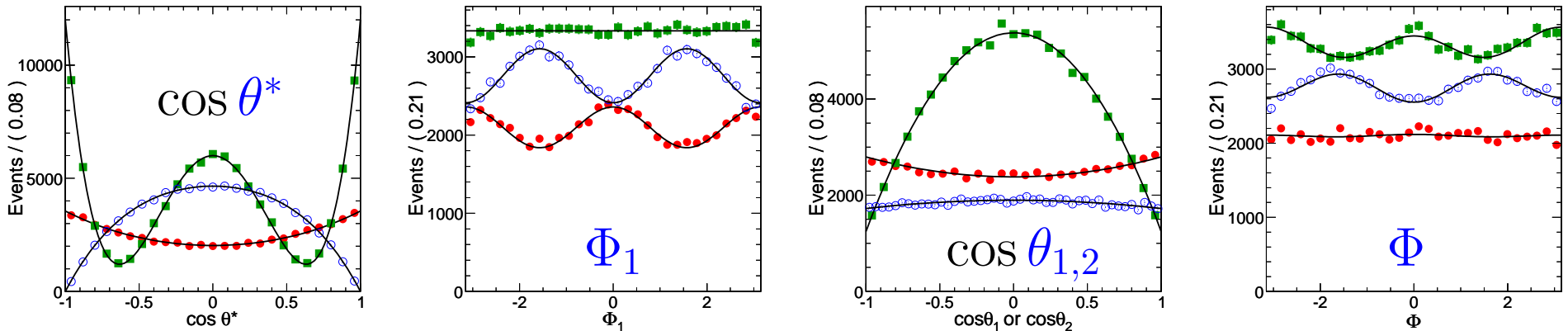
$$r = c_A/c_V \Rightarrow R_{1,2} = 2r_{1,2}/(1 + r_{1,2}^2) = 0.15 (l^\pm), 0.67 (u), 0.94 (d)$$

More Distribution Examples

- Vector 1^- (b_1) and 1^+ (b_2) at $m_H = 250$ GeV
 - lines from derived distributions, points from MC



- G 2_m^+ (minimal), 2_L^+ (Higgs-like), and 2^- ($c_{5,6}$) at $m_H = 250$ GeV



Explicit Distributions for any J

- $d\Gamma(ab \rightarrow X_J \rightarrow Z_1 Z_2^{(*)} \rightarrow (f_1 \bar{f}_1)(f_2 \bar{f}_2)) \propto$
 $F_{00}^J(\theta^*) \times \left\{ \begin{aligned} &4 f_{00} \sin^2 \theta_1 \sin^2 \theta_2 + (f_{++} + f_{--}) ((1 + \cos^2 \theta_1)(1 + \cos^2 \theta_2) + 4R_1 R_2 \cos \theta_1 \cos \theta_2) \\ &- 2(f_{++} - f_{--}) (R_1 \cos \theta_1 (1 + \cos^2 \theta_2) + R_2 (1 + \cos^2 \theta_1) \cos \theta_2) \\ &+ 4\sqrt{f_{++} f_{00}} (R_1 - \cos \theta_1) \sin \theta_1 (R_2 - \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{++}) \\ &+ 4\sqrt{f_{--} f_{00}} (R_1 + \cos \theta_1) \sin \theta_1 (R_2 + \cos \theta_2) \sin \theta_2 \cos(\Phi - \phi_{--}) \\ &+ 2\sqrt{f_{++} f_{--}} \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Phi + \phi_{++} - \phi_{--}) \end{aligned} \right\} \quad \text{spin} = 0 \ \& \ \geq 2$
- $+4F_{11}^J(\theta^*) \times \left\{ \begin{aligned} &(f_{+0} + f_{0-})(1 - \cos^2 \theta_1 \cos^2 \theta_2) - (f_{+0} - f_{0-})(R_1 \cos \theta_1 \sin^2 \theta_2 + R_2 \sin^2 \theta_1 \cos \theta_2) \\ &+ 2\sqrt{f_{+0} f_{0-}} \sin \theta_1 \sin \theta_2 (R_1 R_2 - \cos \theta_1 \cos \theta_2) \cos(\Phi + \phi_{+0} - \phi_{0-}) \end{aligned} \right\}$
- $+4F_{-11}^J(\theta^*) \times (-1)^J \times \left\{ \begin{aligned} &(f_{+0} + f_{0-})(R_1 R_2 + \cos \theta_1 \cos \theta_2) - (f_{+0} - f_{0-})(R_1 \cos \theta_2 + R_2 \cos \theta_1) \\ &+ 2\sqrt{f_{+0} f_{0-}} \sin \theta_1 \sin \theta_2 \cos(\Phi + \phi_{+0} - \phi_{0-}) \end{aligned} \right\} \sin \theta_1 \sin \theta_2 \cos(2\Psi) \quad \text{spin} = 1 \ \& \ \geq 2$
- $+2F_{22}^J(\theta^*) \times f_{+-} \left\{ (1 + \cos^2 \theta_1)(1 + \cos^2 \theta_2) - 4R_1 R_2 \cos \theta_1 \cos \theta_2 \right\}$
- $+2F_{-22}^J(\theta^*) \times (-1)^J \times f_{+-} \sin^2 \theta_1 \sin^2 \theta_2 \cos(4\Psi) \quad \text{spin} \geq 2 \ \text{unique}$
- $+ \text{other 26 interference terms for spin} \geq 2$

where $\Psi = \Phi_1 + \Phi/2$ and $F_{ij}^J(\theta^*) = \sum_{m=0, \pm 1, \pm 2} f_m d_{mi}^J(\theta^*) d_{mj}^J(\theta^*)$

Detector Effects

- Detector effects shape angular distributions (CMS as a reference):

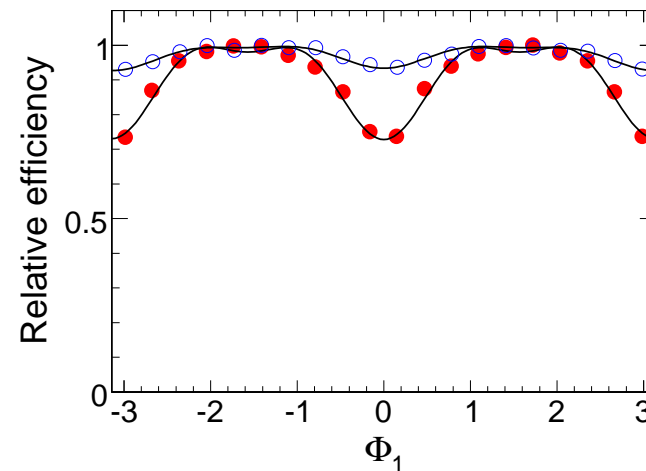
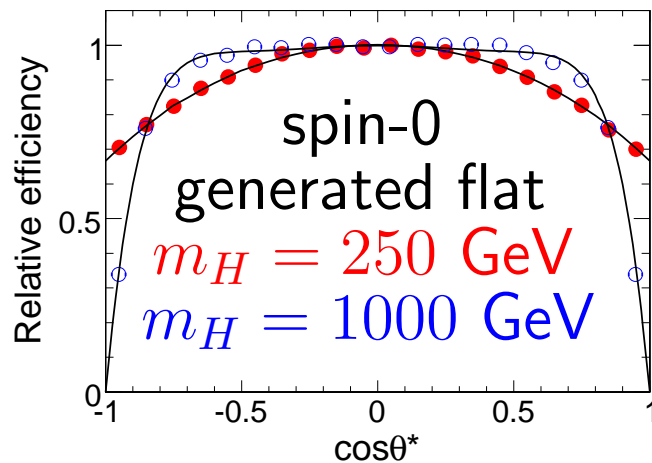
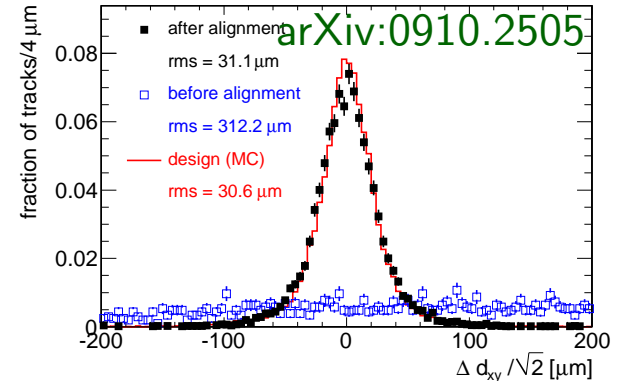
(1) track parameter resolution

⇒ ± 0.01 rad angles

± 3.5 GeV mass at 250 GeV

(2) loss of tracks at $\theta_{\text{lab}} < \theta_{\text{min}}$ ($\eta_{\text{max}} = 2.5$)

(along the beampipe)



– major effect to account for in analysis

acceptance function $\mathcal{G}(\Phi_1, \theta^*, \theta_1, \theta_2, \Phi; Y_X)$

- Fast MC: reject tracks and smear track parameters

Data Analysis (shown with MC)

Analysis Goals

- Analysis depends on how we ask the **question**:

(0) hypothesis **h1** fit self-consistency, goodness-of-fit, χ^2 test, etc...

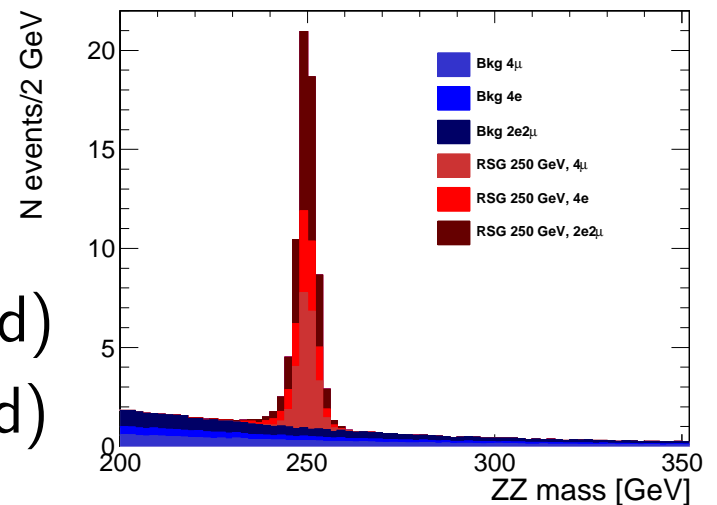
(1) compare hypotheses **h1** and **h2**: confidence in **one** vs **the other**

example (A): **h1**: signal + background

h2: only background

example (B): **h1**: signal 0^+ (+ background)

h2: signal 0^- (+ background)



(2) determine **all parameters** at once (ultimately the best one can do)

yield, mass, width

spin (J)

coupling constants (amplitudes $A_{\lambda_1 \lambda_2}$)

production mechanism (initial polarization f_{zm})

Multivariate Maximum Likelihood Fit

- Maximize likelihood \mathcal{L} (RooFit/MINUIT, from $B \rightarrow VV$):
(*BABAR* PRD78,092008(2008))

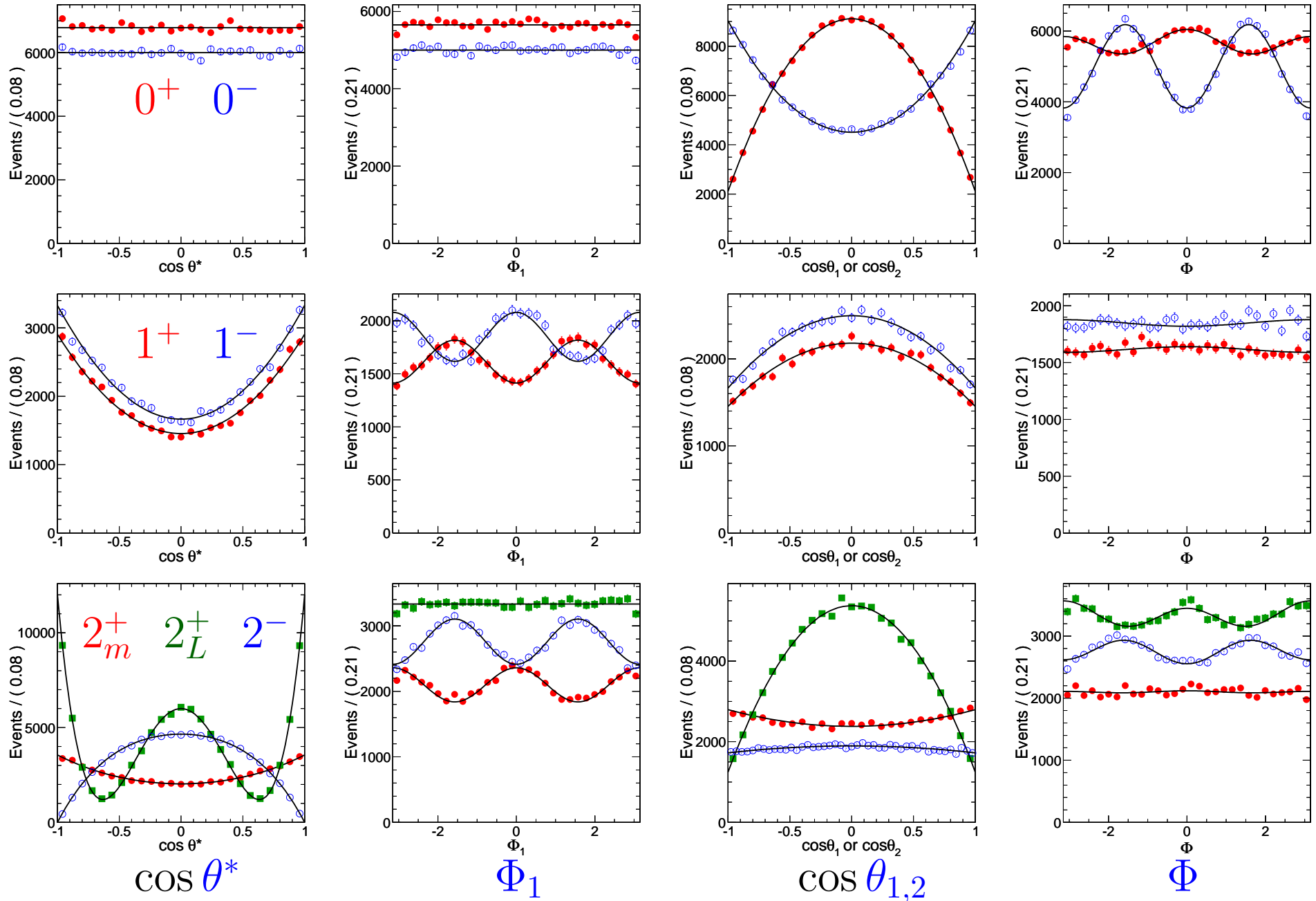
$$\mathcal{L} = \exp\left(-\sum_{J=1}^3 n_J - n_{\text{bkg}}\right) \prod_i^N \left(\sum_{J=1}^3 n_J \times \mathcal{P}_J(\vec{x}_i; \vec{\zeta}_J; \vec{\xi}) + n_{\text{bkg}} \times \mathcal{P}_{\text{bkg}}(\vec{x}_i; \vec{\xi}) \right)$$

$$\vec{\zeta}_J = (f_{\lambda_1\lambda_2}, \phi_{\lambda_1\lambda_2}, f_{zm}; m_X, \Gamma_X), \text{ float } n_J, \text{ fix or float } m_X, \Gamma_X$$

$$\vec{x}_i = (\theta^*, \Phi_1, \theta_1, \theta_2, \Phi; m_{ZZ}, \dots)$$

- Probability \mathcal{P} : (a) template (fixed multi-D histogram)
(b) $\mathcal{P}_J = \mathcal{P}(m_{ZZ}, \dots) \times \mathcal{P}_{\text{ideal}}(\theta^*, \Phi_1, \theta_1, \theta_2, \Phi) \times \mathcal{G}(\theta^*, \Phi_1, \theta_1, \theta_2, \Phi; Y_X)$
- Our choice (b) \Leftarrow both approaches (1) and (2) possible:
 - (1) compare \mathcal{L}_1 vs \mathcal{L}_2 with parameters fixed ($f_{\lambda_1\lambda_2}, \phi_{\lambda_1\lambda_2}, f_{zm}$)
 - (2) fit for all parameters ($f_{\lambda_1\lambda_2}, \phi_{\lambda_1\lambda_2}, f_{zm}$)

Distribution Examples (θ^* , Φ_1 , θ_1 , θ_2 , Φ)

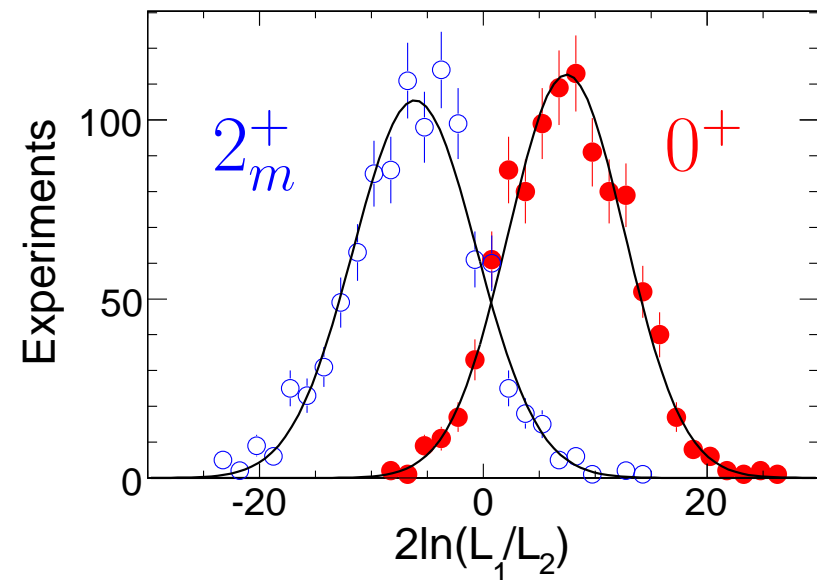
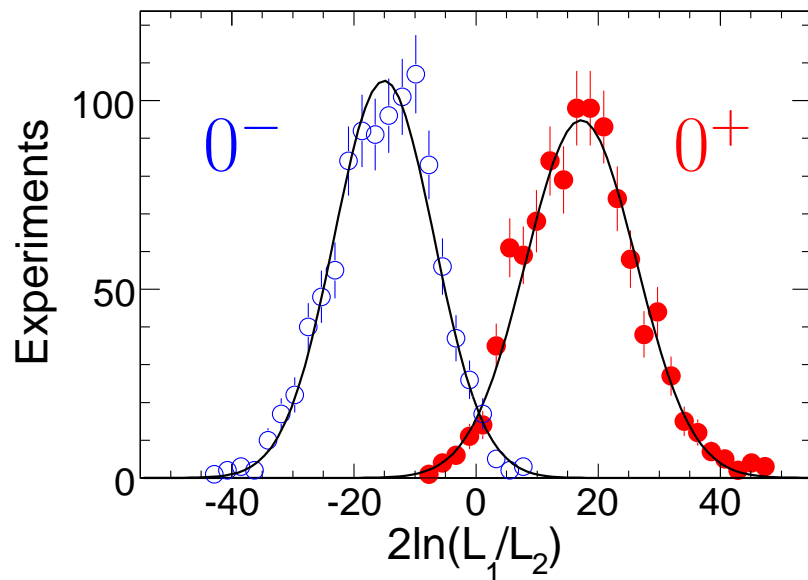


Analysis Approach (1)

- Pick a test scenario with Higgs $m_X = 250$ GeV
 - signal soon after discovery \Rightarrow 30 events (SM Higgs rate)
 - 24 background ($m_{ZZ} = 250 \pm 20$ GeV, $\mathcal{L} = 5/\text{fb}$, $E_{pp} = 14$ TeV)
 - significance 5.7σ signal/background; $\sim 20\%$ gain with angles
- Generate experiments 1000 times
 - plot $2 \ln(\mathcal{L}_1/\mathcal{L}_2)$ for h1 and h2
 - \mathcal{S} effective separation of peaks (Gaussian σ)

$$\mathcal{S} = 4.1\sigma$$

$$\mathcal{S} = 2.8\sigma$$



Analysis Approach (1): Results

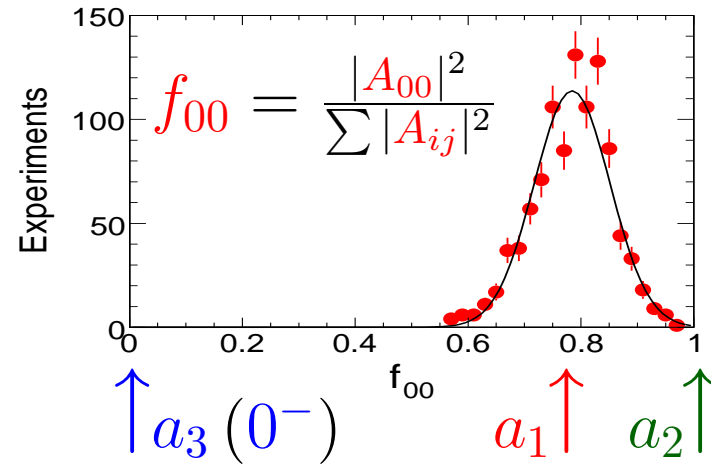
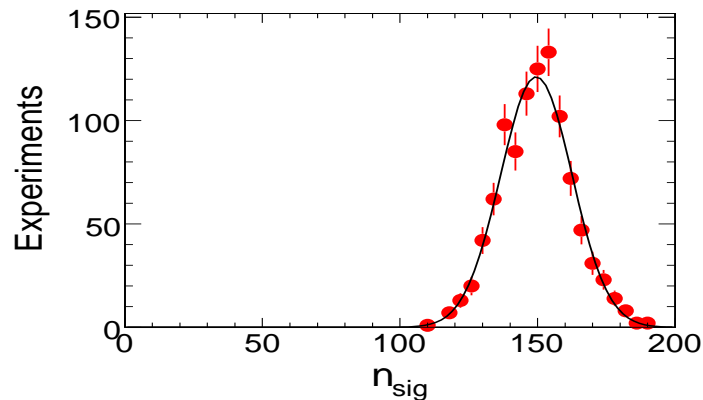
- Example of separation at $m_X = 250$ GeV (similar at 1000 GeV)
 - with 30 events $\sim 2 - 4\sigma$ separation
 - full event info (production+decay) \Rightarrow ultimate precision

1D (θ^*) / 3D (θ_1, θ_2, Φ) / 5D ($\Phi_1, \theta^*, \theta_1, \theta_2, \Phi$)

	0^-	1^+	1^-	2_m^+	2_L^+	2^-
0^+	0.0/3.9/4.1	0.8/1.8/2.3	0.9/2.5/2.6	0.8/2.4/2.8	2.6/0.0/2.6	1.6/2.4/3.3
0^-	–	0.8/2.8/3.1	0.9/2.5/3.0	0.8/1.7/2.4	2.9/4.1/4.8	1.6/2.0/2.9
1^+	–	–	0.0/1.1/2.2	0.1/1.3/2.6	2.8/1.9/3.6	2.5/1.2/2.9
1^-	–	–	–	0.1/1.3/1.8	2.8/2.5/3.8	2.5/0.6/3.4
2_m^+	–	–	–	–	2.9/2.6/3.8	2.3/0.5/3.2
2_L^+	–	–	–	–	–	3.6/2.5/4.3

Analysis Approach (2)

- More general approach: fit all parameters (spin-0: Higgs 250 GeV)
 - $\times 5$ more events (150 signal & 120 background)



	generated	w/o detector	with detector
n_{sig}	150	150 ± 13	153 ± 15
f_{00}	0.792	0.79 ± 0.07	0.77 ± 0.08
$(f_{++} - f_{--})/2$	0.000	0.00 ± 0.07	0.01 ± 0.07
$(\phi_{++} + \phi_{--})/2$	π	3.15 ± 0.73	3.20 ± 0.77
$(\phi_{++} - \phi_{--})/2$	0	0.00 ± 0.53	0.01 ± 0.55

- Tested all 7 hypotheses at $m_X = 250$ and 1000 GeV

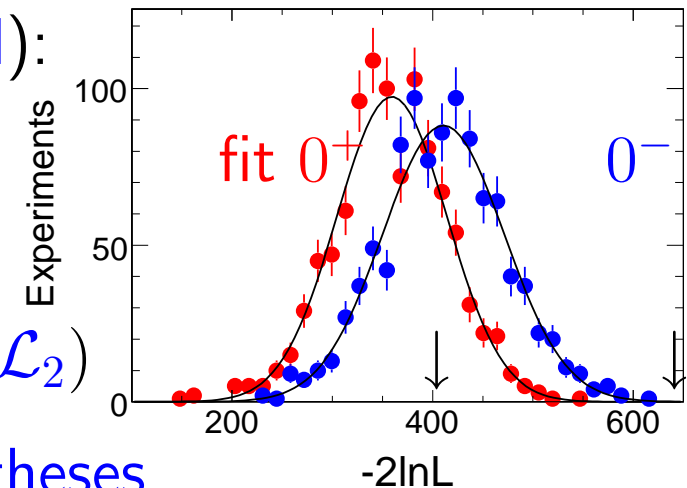
Higgs or not Higgs?

- Three steps in analysis (include **mass**, **yield**):

(1) hypothesis **h1** fit self-consistency \rightarrow
" χ^2 " = $-2 \ln(\mathcal{L}_1)$ from background & signal

(2) compare hyp. **h1** and **h2** with $2 \ln(\mathcal{L}_1/\mathcal{L}_2)$

(>2) determine **all parameters** for **all hypotheses**...



- If found resonance is **not truly SM Higgs**

and parameters are rather different \Rightarrow exclude SM Higgs

\Rightarrow quote "**range**" of allowed hypotheses

- If **true SM Higgs** is found

can we exclude **all other hypotheses**?

\Rightarrow only very fine-tuned hypotheses cannot be ruled out "easily"

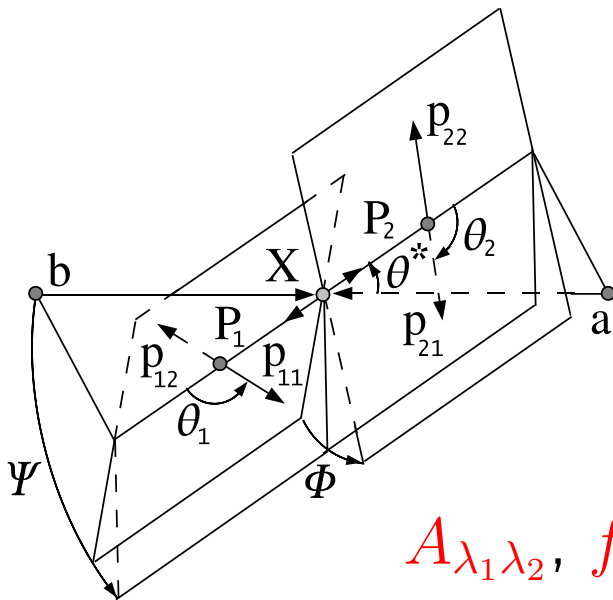
e.g. **unpolarized** "graviton" with Higgs-like **couplings**, **rate**, **width**...

\Rightarrow quote level of **consistency** and "**range**" of excluded hypotheses

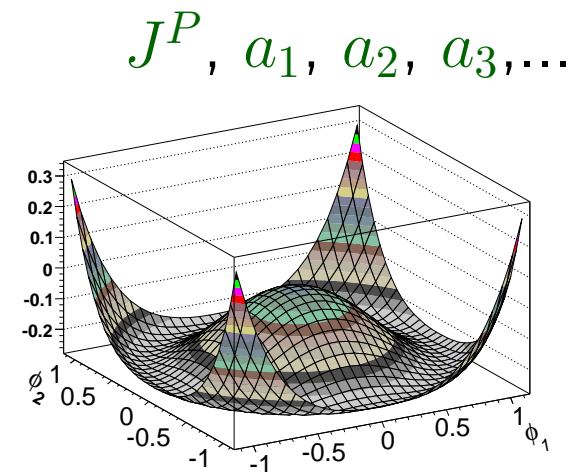
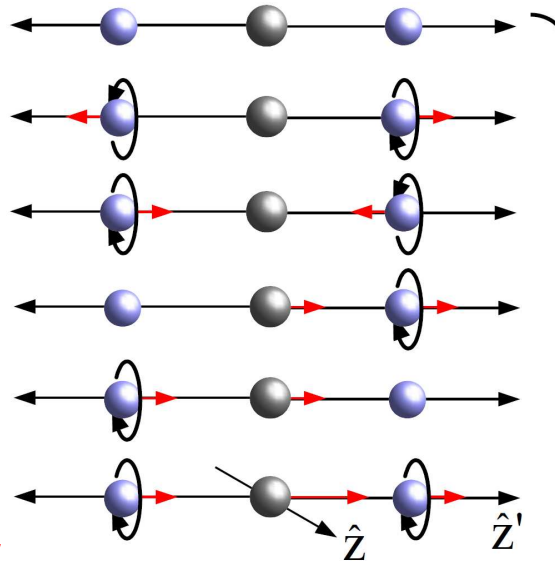
Conclusion

Conclusion

- Resonances at LHC: either **within** (Higgs) or **beyond SM**
 - maximum info \Rightarrow **spin**, **quantum numbers**, **couplings**
 - powerful angular technique, example $X_J \rightarrow ZZ/WW$
 - \rightarrow combine **production** and **decay** angles
 - \rightarrow **MC**, **angular distributions**, **ML fit** \rightarrow **3-4 σ** soon after discovery
 - helicity formalism ("**exp.**") \leftrightarrow quantum n. & couplings ("**theory**")
 - model-independent approach (!)



$$A_{\lambda_1 \lambda_2}, f_{zm}$$



BACKUP

$X \rightarrow ZZ$ polarization notation

- Polarization notation:

$$e_1^\mu(\lambda_1 = 0) = \left(\frac{\beta M_X}{2M_V}, 0, 0, \frac{M_X}{2M_V}\right) \quad \perp \quad q_1^\mu = \left(\frac{M_X}{2}, 0, 0, \frac{\beta M_X}{2}\right)$$

$$e_1^\mu(\lambda_1 = \pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

$$t^{\mu\nu}(J_{z'} = +2) = e_X^\mu(+)\epsilon_X^\nu(+), \text{ etc...}$$

- Amplitude with field strength tensor $F^{\mu\nu}$ (e.g. **graviton couplings**):

$$\begin{aligned} A(X_{J=2} \rightarrow VV) = \Lambda^{-1} & \left[2g_1^{(2)} t_{\mu\nu} F^{*1,\mu\alpha} F^{*2,\nu\alpha} + 2g_2^{(2)} t_{\mu\nu} \frac{q_\alpha q_\beta}{\Lambda^2} F^{*1,\mu\alpha} F^{*2,\nu,\beta} \right. \\ & + g_3^{(2)} \frac{\tilde{q}^\beta \tilde{q}^\alpha}{\Lambda^2} t_{\beta\nu} (F^{*1,\mu\nu} F_{\mu\alpha}^{*2} + F^{*2,\mu\nu} F_{\mu\alpha}^{*1}) + g_4^{(2)} \frac{\tilde{q}^\nu \tilde{q}^\mu}{\Lambda^2} t_{\mu\nu} F^{*1,\alpha\beta} F_{\alpha\beta}^{*(2)} \\ & + m_V^2 \left(2g_5^{(2)} t_{\mu\nu} \epsilon_1^{*\mu} \epsilon_2^{*\nu} + 2g_6^{(2)} \frac{\tilde{q}^\mu q_\alpha}{\Lambda^2} t_{\mu\nu} (\epsilon_1^{*\nu} \epsilon_2^{*\alpha} - \epsilon_1^{*\alpha} \epsilon_2^{*\nu}) + g_7^{(2)} \frac{\tilde{q}^\mu \tilde{q}^\nu}{\Lambda^2} t_{\mu\nu} \epsilon_1^* \epsilon_2^* \right) \\ & + g_8^{(2)} \frac{\tilde{q}_\mu \tilde{q}_\nu}{\Lambda^2} t_{\mu\nu} F^{*1,\alpha\beta} \tilde{F}_{\alpha\beta}^{*(2)} + g_9^{(2)} t_{\mu\alpha} \tilde{q}^\alpha \epsilon_{\mu\nu\rho\sigma} \epsilon_1^{*\nu} \epsilon_2^{*\rho} q^\sigma \\ & \left. + \frac{g_{10}^{(2)} t_{\mu\alpha} \tilde{q}^\alpha}{\Lambda^2} \epsilon_{\mu\nu\rho\sigma} q^\rho \tilde{q}^\sigma (\epsilon_1^{*\nu} (q\epsilon_2^*) + \epsilon_2^{*\nu} (q\epsilon_1^*)) \right] \end{aligned}$$